Assignment 7 - Violating Fairness Criteria in Voting Rubric

# Information

In this assignment, you will **NOT** be coding anything!!! Instead, we will take this opportunity to see how Arrow’s Theorem (mentioned at the end of our [reading](https://docs.google.com/document/d/1RZH0Z3-yBIjCSIaX1vf0LP5PZntR60Xx3PIHfJImT-U/edit?usp=sharing)) comes into play with some of our voting methods. In particular, the theorem points to four different criteria for the “fairness” of an election, and states that it is impossible to satisfy all four in every election under the assumption that the elections are held using preference schedules. We will look at each of the four criteria and develop elections that violate them.

# Tasks

Your tasks for this assignment are to design a preference schedule for each of the four “fairness criteria” such that each criterion fails. For each of these examples, we will (by default) assume a 10-voter election with (by default) three candidates: Adelaide, Bronson, and Cordelia.

1. **The Majority Criterion**: *If a candidate receives a majority of the 1st-place votes in an election, then that candidate should be the winner of the election*.

Develop a preference schedule such that the Borda method produces a different winner than the Majority rule. This election is an example of one which violates the Majority Criterion. If you need to alter the number of voters or candidates to make your example work, you are free to do so.

| **V1** | **V2** | **V3** | **V4** | **V5** | **V6** | **V7** | **V8** | **V9** | **V10** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | A | A | A | A | A | B | B | B | B |
| B | B | B | B | B | B | C | C | C | C |
| C | C | C | C | C | C | A | A | A | A |

**A wins by Majority, B wins by Borda.**

1. **The Condorcet Criterion**: *If there is a candidate that in a head-to-head comparison is preferred by the voters over every other candidate, then that candidate should be the winner of the election. This candidate is known as the Condorcet candidate*.

Develop a preference schedule such that the Borda method produces a winner that is not a Condorcet winner. This election is an example of one which violates the Condorcet Criterion. If you need to alter the number of voters or candidates to make your example work, you are free to do so.

| **V1** | **V2** | **V3** | **V4** | **V5** | **V6** | **V7** | **V8** | **V9** | **V10** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | A | A | A | A | A | B | B | B | B |
| B | B | B | B | B | B | C | C | C | C |
| C | C | C | C | C | C | A | A | A | A |

The previous schedule also works here. In this case, A wins pairwise elections with B and C 6/11 times, but the Borda scores are A: 22, B: 24, C: 14, so B wins but is not a Condorcet winner.

1. **The Monotonicity Criterion**: *If candidate X is a winner of an election and, in a re-election, the only changes in the ballots are changes that favor X, then X should remain a winner of the election.*

For this exercise, you will develop two preference schedules. The first will represent the initial election, and the second will represent the election after some ballots have been changed. The election will proceed using the Hare method. The condition for these preference schedules is that, if the winner of the first election is candidate X, then the changes to the ballots can *only* improve the relative position of X. However, the changes must be made in such a way that boosting X causes them to lose. This election is an example of one which violates the Monotonicity Criterion. If you need to alter the number of voters or candidates to make your example work, you are free to do so.

*Schedule 1*

| **# Voters** | **14** | **4** | **8** | **11** |
| --- | --- | --- | --- | --- |
|  | A | B | B | C |
|  | B | A | C | A |
|  | C | C | A | B |

*Schedule 2*

| **# Voters** | **16** | **2** | **8** | **11** |
| --- | --- | --- | --- | --- |
|  | A | B | B | C |
|  | B | A | C | A |
|  | C | C | A | B |

In the first preference schedule, C is eliminated first, giving A 25 votes and B 12 votes. In the second schedule, the only change that was made was that two voters switched from B>A>C to A>B>C - an increase to A’s position. This causes B to get eliminated first, leaving A with 18 votes and C with 19, so C wins.

1. **The Independence of Irrelevant Alternatives Criterion**: *If candidate X is a winner of an election and one (or more) of the other candidates is removed and the ballots recounted, then X should still be a winner of the election*.

Develop a preference schedule such that there is a candidate X who would win an election decided by pairwise comparisons, but would lose if another candidate Y is removed from the race, leaving the relative order of the other two candidates on the ballots unchanged. For this method, when comparing candidate X to candidate Y, give X 1 point if it is ranked higher than Y in the majority of the votes, 0 points if it is ranked lower than Y in the majority of the votes, and ½ point if they are evenly split. Do these comparisons between each pair of candidates and add up each candidate’s points at the end. The candidate with the most points wins. If you need to alter the number of voters or candidates to make your example work, you are free to do so.

| **V1** | **V2** | **V3** | **V4** | **V5** | **V6** | **V7** | **V8** | **V9** | **V10** | **V11** | **V12** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | A | A | B | B | C | C | C | C | D | D | D |
| C | C | C | D | D | B | B | B | B | A | A | A |
| B | B | B | A | A | D | D | D | D | C | C | C |
| D | D | D | C | C | A | A | A | A | B | B | B |

A: .5 + 1 = 1.5

B: .5 + 1 = 1.5

C: 1 + 1 = 2

D: 1 = 1

With D in the election, C wins with 2 points.

| **V1** | **V2** | **V3** | **V4** | **V5** | **V6** | **V7** | **V8** | **V9** | **V10** | **V11** | **V12** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | A | A | B | B | C | C | C | C | A | A | A |
| C | C | C | A | A | B | B | B | B | C | C | C |
| B | B | B | C | C | A | A | A | A | B | B | B |

A: .5 + 1 = 1.5

B: .5 = .5

C: 1 = 1

When D is removed from the election, A wins with 1.5 points. Nothing else about the ballots changed - all the relative orders of the other candidates are still intact.

Once you have completed all of the tasks above, submit a link to this document to Moodle (make sure Sharing is turned **ON**).